M.Phil/Pre Ph.D Regular & Supplementary Examinations – OCTOBER, 2023
R211002
Paper –II: MATHEMATICAL METHODS.
MATHEMETICS

Time : 3 hrs Maximum Marks : 100

Answer One Question from Each Unit
All Questions Carry Equal Marks

UNIT-I

1. a) If u is harmonic, show that $f = u_x - iu_y$ is analytic.
   b) Define Green's function and find the Green's function for the region $G = \{z: Re\, z > 0\}$.

(OR)

2. a) Prove that a harmonic function is an open map.
   b) Let f be an entire function of finite order, then show that f assumes each complex number with one possible exception.

UNIT-II

3. a) Write about physical significance of stream function?
   b) Show that $u = 2Cxy, v = C(a^2 + x^2 - y^2)$ are given velocity components of a possible fluid motion. Determine the stream function.

(OR)

4. a) A two-dimensional flow field is given by $\Psi = xy$. Show that the flow is irrotational and also find the velocity potential?
   b) λ is denoting a variable parameter, and f is a given function, find the condition that $f(x, y, \lambda) = 0$ should be a possible system of stream lines for steady irrotational motion in two dimensions.

UNIT-III

5. a) Find the solution of the Laplace equation $u_{xx} + u_{yy} = 0$ in the given region R, where R is a square of side 3 units. Subject to the boundary conditions $u(0, y) = 0$, $u(3, y) = 3 + y,\ u(x, 0) = x, u(x, 3) = 2x$. Assume step length as $h = 1$.
   b) Solve $u_{xx} = 32u_t$, $0 \leq x \leq 1$,taking $h = 0.5$and $u(x, 0) = 0$, $0 \leq x \leq 1$, $u(0, t) = 0, u(1, t) = t,\ t > 0$. Use an explicit method with $\lambda = 1/2$. Compute for four time steps.

(OR)

6. a) Find the solution of the Poisson's equation $u_{xx} + u_{yy} = G(x, y)$ in the given region R, where $R: 0 \leq x \leq 1, 0 \leq y \leq 1. G(x, y) = 4, u(x, y) = x^2 + y^2$ on the boundary . Assume step length as $h = 1/3$.
   b) Solve the boundary value problem $xy" + y = 0$, $y(1) = 1, y(2) = 2$ by second order finite difference method with $h = 0.25$.

UNIT-IV

7. a) Find a one-parameter Galerkin solution of the equation $-\nabla^2 u = 1$ in Ω (=unit square); $u = 0$ on Γ. Use algebraic approximation function.

   b) Consider the differential equation $-\frac{d^2u}{dx^2} = \cos \pi x$ for$0 < x < 1$, subject to the boundary conditions $u(0) = 0, u(1) = 0$. Determine the three parameter solution, with trigonometric functions, using the Rayleigh-Ritz method.

(OR)

8. a) Find a one-parameter Rayleigh-Ritz solution of the equation $-\nabla^2 u = 1$ in Ω (=unit square); $u = 0$ on Γ. Use algebraic approximation function.

   b) Find the weak form of the equation $-\frac{d}{dx}\left(u\frac{du}{dx}\right) + f = 0$ for $0 < x < 1$ subject to the condition $\left(\frac{du}{dx}\right)\Big|_{x=0} = 0,\ \ u(1) = \sqrt{2}$.

UNIT-V

9. a) Minimize the quantity $f = x^2 + y^2 + z^2$ subject to the surfaces $z = xy + 5$ *and* $x + y + z = 1$ by using Lagrange multipliers.

   b) Transform the problem $\frac{d^2y}{dx^2} + xy = 1$, $y(0) = y(1) = 0$ to the integral equation $y(x) = \int_0^1 G(x,\xi)\xi y(\xi)d\xi - \frac{1}{2}x(1-x)$, where $G(x,\xi) = x(1-\xi)$ when $x < \xi$ and $G(x,\xi) = \xi(1-x)$ when $x > \xi$.

(OR)

10. a) Of all parabolas which pass through the points $(0,0)$ and $(1,1)$, determine that one which, when rotated about the x axis, generates a solid of revolution with least possible volume between $x = 0$ *and* $x = 1$. (The equation may be taken in the form $y = x + cx(1-x)$, where 'c' is to be determined).

    b) Show that, if $y(x)$ satisfies the differential equation $\frac{d^2y}{dx^2} + xy = 1$ and the conditions $y(0) = y'(0) = 0$, then y also satisfies the Volterra equation

    $y(x) = \int_0^x \xi(\xi - x)y(\xi)d\xi + \frac{1}{2}x^2$.

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